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Dual Connected Neighborhood Domination In Graphs

V. Mohanaselvi[1], K. Deiwakumari[2], S. Dhivyakannu[3]

Abstract— A subset D ⊆ V (G) of a graph G = (V, E) is said to be **dual connected neighborhood domination set** of G if D is connected domination set of G and N (G). The **dual connected neighborhood domination number** is the minimum cardinality taken over all connected neighborhood dominating sets of G and is denoted by $\boldsymbol{\gamma_{dcn}(G)}$. In this paper, $\gamma_{dcn}(G)$ are obtained for some standard graphs.

Index Terms— Connected domination number, Global domination number, Global neighborhood domination number.

1 Introduction

In this paper G = (V, E) a finite, simple, connected and undirected graph has p-vertices and q-edges. Terms not defined here are used in the sense of Harary [2].

The *neighborhood graph N(G)* of a graph G is the graph having the same vertex set as G with two vertices are adjacent in *N(G)* if and only if they have a common neighbor in G. This was introduced by R.C. Brigham and D. Dutton in [7].

Degree of a vertex v is denoted by d(v), the maximum(minimum)degree of a graph G is denoted by $\Delta(G)(\delta(G))$. A vertex v is said to be isolated vertex if d(v) = 0.

A set D of vertices of a graph G = (V, E) is a *dominating set* of G if every vertex in V-D is adjacent to some vertex in D. The *domination number* $\boldsymbol{\gamma(G)}$ of G is the minimum cardinality of all dominating sets of G. This concept was introduced by Ore in [1].

The concept of connected domination number was introduced by E. Sampathkumar and Walikar in [3]. A set D ⊆ V(G) is said to be *connected dominating set*, if the induced subgraph <D> is connected. The *connected domination number* $\boldsymbol{\gamma_c(G)}$ of a connected graph G is the minimum cardinality of a connected dominating set of G.

In [4], E.Sampathkumar introduced the concept of global domination number as follows: A set D ⊆ V(G) is said to be *global dominating set*, if D is a dominating set of G and $\bar{G}$.The *global domination number* $\boldsymbol{\gamma_g(G)}$ is the minimum cardinality of a global dominating set of G.

A subset D of vertices of a graph G is called a *global neighborhood dominating set* (gnd – set) if D is a dominating set for both G and N(G), where N(G) is the neighborhood graph of G. The global neighborhood domination number (gnd - number) is the minimum cardinality of a global neighborhood dominating set of G and is denoted by $\boldsymbol{\gamma_{gn}(G)}$. This concept was introduced by S. V. Siva Rama Raju and I. H. Nagaraja Rao in [8].

In this paper, we introduced dual connected neighborhood domination by combining the concept of connected domination and global neighborhood domination for a connected graph. The characteristic was studied and the exact value of dual connected neighborhood domination was found for some standard graphs and bounds. The characteristics was studied and the exact.

- *Dr. V. Mohanaselvi, Assistant Professor of Mathematics, Nehru Memorial College, Puthanampatti, Tiruchirappalli-621007.E-mail: vmohanaselvi@gmail.com*
- *S. Divyakannu, Lecture of Mathematics, Nehru Memorial College, Puthanampatti,Tiruchirappalli-621007. E-mail: divkannu@gmail.com*

2. Main Results

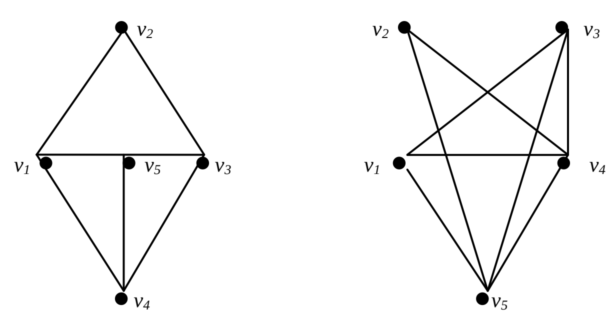
2.1. Dual Connected Neighborhood Domination Number

Definition 2.1.1

A vertex set D ⊆ V(G) of a graph G = (V, E) is said to be **dual connected neighborhood dominating set**(dcnd-set) of G if D is connected domination set of G and dominating set of N(G). The dual **connected neighborhood domination number** is the minimum cardinality taken over all dual connected neighborhood dominating sets of G and is denoted by $\boldsymbol{\gamma_{dcn}(G)}$. A dcnd-set of G with minimum cardinality is denoted by γ_{dcn} - set of G.

Throughout this chapter, we assume that G is connected.

Example 2.1.1



G Figure N(G)

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For the graph G in figure, the vertex set D = $\{v_1, v_5\}$ is a γ_{dcn}-set and hence $\gamma_{dcn}(G)$ = 2.

2.2 BOUNDS AND RESULTS ON $\gamma_{dcn}(G)$

The following theorem gives the relation between domination, connected domination and global neighborhood domination for a graph G.

Theorem: 2.2.1

For any graph G, $\gamma(G) \leq \gamma_{gn}(G) \leq \gamma_{\mathrm{dcn}}(G)$

Proof:

Since every global neighborhood dominating set is a dominating of G,

We have $\gamma(G) \leq \gamma_{gn}(G)$ …(1)

Also, every dual connected neighborhood dominating set is a global neighborhood dominating set of G and hence

$\gamma_{gn}(G) \leq \gamma_{\mathrm{dcn}}(G)$ …(2)

From (1) & (2), the result follows.

For example the graph G in figure, the vertex set D=$\{v_1, v_5\}$ is a γ_{dcn}-set. So, $\gamma_{dcn}(G)$ = 2.
Since <D> is global neighborhood, $\gamma_{gn}(G)$ = 2. Therefore, we have $\gamma_{gn}(G) = \gamma_{dcn}(G)$.

The exact values of dual connected neighborhood domination number $\gamma_{dcn}(G)$ for some standard graphs are given below.

Theorem: 2.2.2

For the complete graph K_n, $\gamma_{\mathrm{dcn}}(K_n) = 2$, n ≥ 2.

Proof:

Let G be the complete graph K_n with at least one vertex and $V(G) = \{v_1, v_2, \dots, v_n\}$ is a vertex set of G. Since *N(G)* is also a complete graph. Therefore, the γ_{dcn} - set = *2*.

Hence $\gamma_{dcn}(G) = 2$.

Theorem: 2.2.3

For the wheel graph W_n, $\gamma_{\mathrm{dcn}}(W_n) = 2$, n ≥ 3.

Proof:

Let *G* be a wheel graph W_n with at least 4 vertices and $V(G) = \{u, v_1, v_2, \dots, v_n\}$ be the vertex set of *G*. Therefore, $|V(G)| = 2$.

Since neighborhood of G is a complete. Hence, $\gamma_{dcn}(G) \leq |V(G)| = 2$.

Theorem: 2.2.4

For the fan graph $F_{1,n}$, $\gamma_{\mathrm{dcn}}(F_{1,n}) = 2$,for n ≥ 2.

Proof:

Let G be a fan graph $F_{1,n}$ with atleast 3 vertices and V (G) = $\{u, v_1, v_2, v_3, \dots, v_n\}$ be the vertex set of G.

Let $u \in V(G)$ has the maximum degree in G and v_i be any vertex adjacent to u in G. Then the set $S = \{u, v_i\}$ forms a dual connected neighborhood set of G.

Hence $\gamma_{\mathrm{dcn}}(G) \leq |S| = 2$ …(1)

Let S be the γ_{dcn} - set of G. Dual connected neighborhood dominating set in N(G) must contain one maximum degree vertex in N(G). Hence the dual connected neighborhood set has at least 2 vertices. $\gamma_{\mathrm{dcn}}(G) = |S| \geq 2$ … (2)

Then, the result follows from (1) and (2).

Theorem: 2.2.5

For the friendship graph, $\gamma_{\mathrm{dcn}}(C_3^m) = 2$, for m ≥ 1.

Proof:

Let G be a friendship graph with at least 3 vertices and V (G) = $\{u, v_1, v_2, v_3, \dots, v_n\}$ be a vertex set of G.

Let $u \in V(G)$ has the maximum degree in G, and v_i be any vertex adjacent to u in G. Then the set $S = \{u, v_i\}$ forms a dual connected neighborhood set of G.

Hence $\gamma_{\mathrm{dcn}}(G) \leq |S| = 2$ …(1)

Let S be the γ_{dcn}-set of G. Dual connected neighborhood dominating set in N (G) must contain one maximum degree vertex in N (G).

Hence the dual connected neighborhood set has at least 2 vertices.

$\gamma_{\mathrm{dcn}}(G) = |S| \geq 2$ … (2)

Then, the result follows from (1) and (2).

Theorem: 2.2.6

For the prism graph C_3XP_n, $\gamma_{\mathrm{dcn}}(C_3 \times P_n) = n$, for n ≥ 2.

Proof:

Let G be a prism graph with at least 6 vertices and V (G) = $\{v_1, v_2, v_3, \dots, v_{3n}\}$ be a vertex set of G. The vertex set $S = \{P_i, i = 1,2 \dots n\}$ is a dual connected neighborhood set of G.

Hence $\gamma_{\mathrm{dcn}}(G) \leq |S| = n$ …(1)

Let S be the γ_{dcn}-set of G. For the connected neighborhood domination of N (G), S must contain adjacent vertices in N(G). $\gamma_{\mathrm{dcn}}(G) = |S| \geq n$ … (2)

Then, the result follows from (1) and (2).

Theorem: 2.2.7

For Corona graph K_n^+, $\gamma_{\mathrm{dcn}}(K_n^+) = n$, for n ≥ 3.

Proof:

Let G be a corona graph with at least 6 vertices and V (G) = $\{v_1, v_2, v_3, \dots, v_{2n}\}$ be a vertex set of G. The vertex set $S = \{v_i, i = 1,2 \dots n\}$ is a dual connected neighborhood set of G.

Hence $\gamma_{\mathrm{dcn}}(G) \leq |S| = n$ …(1)

Let S be the γ_{dcn}-set of G. For the connected neighborhood domination of N (G), S must contain adjacent vertices in N(G). $\gamma_{\mathrm{dcn}}(G) = |S| \geq n$ … (2)

Then, the result follows from (1) and (2).

Theorem 2.2.8

For the helm graph W_n^+, $\gamma_{dcn}(W_n^+) = n + 1, n \geq 3$.

Proof:

Let G be a helm graph with at least 6 vertices and V(G) = { u, $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ } is the vertex set of G.

Let $u \in V(G)$ such that $d(u) = \Delta(G)$ and the vertex set $\{v_i, i = 1,2 \ldots n\} \cup \{u\}$ forms a dcndn - set in G.

Hence, $\gamma_{dcn}(G) \leq |D| = n + 1$ **…(1)**

Let D be γ_{dcn} - set of G. Then D has a maximum degree vertex with its adjacent vertices in G.

Hence, $\gamma_{dcn}(G) = |D| \geq 1+n = n + 1$ **…(2)**

The result follows from (1) & (2).

Theorem: 2.2.9

For Corona graph $F_{1,n}^+$, $\gamma_{dcn}(F_{1,n}^+) = n+1$, for $n \geq 3$.

Proof:

Let G be a corona graph with at least 6 vertices and V (G) = $\{v_1, v_2, v_3, \ldots, v_{2n}\}$ be a vertex set of G.

The vertex set $S = \{v_i, i = 1,2 \ldots n+1\}$ is a dual connected neighborhood set of G.

Hence $\gamma_{dcn}(G) \leq |S| = n+1$ …(1)

Let S be the γ_{dcn}-set of G. For the connected neighborhood domination of N (G), S must contain adjacent vertices in N(G). $\gamma_{dcn}(G) = |S| \geq n + 1$ … (2)

Then, the result follows from (1) and (2).

Theorem: 2.2.10

For Corona graph $C_3^{(m)+}$, $\gamma_{dcn}(C_3^{(m)+}) = 2m + 1$, for $m \geq 1$.

Proof:

Let G be a corona graph with at least 6 vertices and V (G) = V(G) = { u, $v_1, v_2, \ldots, v_{2m}, u_1, u_2, \ldots, u_{2m+1}$ }be a vertex set of G.

The vertex set $S = \{v_i, i = 1,2 \ldots 2m\} \cup \{u\}$ is a dual connected neighborhood set of G.

Hence $\gamma_{dcn}(G) \leq |S| = 2m + 1$ …(1)

Let S be the γ_{dcn}-set of G. For the connected neighborhood domination of N (G), S must contain adjacent vertices in N(G). $\gamma_{dcn}(G) = |S| \geq 2m + 1$ … (2)

Then, the result follows from (1) and (2).

Theorem: 2.2.11

For the triangular snake graph mC_3, $\gamma_{dcn}(mC_3) = \begin{cases} m & , m = 2 \\ m-1, & m \geq 3 \end{cases}$.

Proof:

Let G be a triangular snake graph with at least 5 vertices and V(G) = $\{v_1, v_2, \ldots, v_{m+1}, u_1, u_2, \ldots, u_m\}$be a vertex set of G.

The vertex set $S = \{v_i, i = 2,3,4, \ldots, m\}$ is a dual connected neighborhood set of G.

Hence $\gamma_{dcn}(G) \leq |S| = m - 1$ …(1)

Let S be the γ_{dcn}-set of G. For the connected neighborhood domination of N (G), S must contain adjacent vertices in N(G). $\gamma_{dcn}(G) = |S| \geq m - 1$ … (2)

Then, the result follows from (1) and (2).

The following theorem gives bound for $\gamma_{dcn}(G)$.

Theorem: 2.2.12

For any graph G, $\gamma_{dcn}(G) \leq 2p - \Delta(G)$

Proof:

Let D be the dcnd-set of G. Since $\langle D \rangle$ is connected and $\gamma_c(G) \leq p - \Delta(G)$.

We have D has at most $p - \Delta(G)$ vertices.

But D is also a connected neighborhood dominating set of N(G), We have D has atmost p vertices and hence $\gamma_{dcn}(G) \leq |D| \leq p - \Delta(G) + p = 2p - \Delta(G)$

$\Rightarrow \quad \gamma_{\overline{cc}}(G) \leq 2p - \Delta(G)$

Theorem 2.2.13

For any graph G, $\gamma_{dcn}(G) \leq p - \Delta(G) + 1$

Proof:

Let D be the dcnd-set of G. Since $\langle D \rangle$ is connected and $\gamma_c(T) \leq p - \Delta(G)$.

We have D has at most $p - \Delta(G)$ vertices. But D is also a dominating set of N (G), We have D has atleast one vertex.

Hence $\gamma_{dcn}(G) \leq |D| \leq p - \Delta(G) + 1 = p - \Delta(G) + 1$

$\Rightarrow \quad \gamma_{dcn}(G) \leq p - \Delta(G) + 1$

Theorem 2.2.14

For any graph G , $\gamma_{dcn}(G) \geq p - q + \delta(G)$.

Proof:

Let D be a γ_{dcn}-set of G. Then there exists a vertex $u \in D$ adjacent only to vertices of D. Thus,

$q \geq |V - D| + deg\, u \geq |V - D| + \delta(G)$.

Hence the result.

Proposition 2.2.15

1. If G is a graph then, $\gamma_{dcn}(G) \leq \gamma(G) + 1$.
2. If G is a graph then, $\gamma_{dcn}(G) \leq \gamma_c(G)$

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3. CONCLUSION

In this paper, we found the exact values of dual connected neighborhood domination number for complete graph, wheel graph, fan graph, prism graph, friendship graph, corona graph, triangular snake graph and some bounds.

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